October 2003 hep-th/0310152

On the Universality of Goldstino Action [1]

Tomoya Hatanaka [2] and Sergei V. Ketov [3]

Department of Physics, Faculty of Science
Tokyo Metropolitan University
1–1 Minami-osawa, Hachioji-shi
Tokyo 192–0397, Japan

Abstract

We find the Goldstino action descending from the N=1 Goldstone-Maxwell superfield action associated with the spontaneous partial supersymmetry breaking, N=2 to N=1, in superspace. The new Goldstino action has higher (second-order) spacetime derivatives, while it can be most compactly described as a solution to the simple recursive relation. Our action seems to be related to the standard (having only the first-order derivatives) Akulov-Volkov action for Goldstino via a field redefinition.

[1] Supported in part by the Japanese Society for Promotion of Science (JSPS) and the TMU fund

[2] Email address: thata@kiso.phys.metro-u.ac.jp
[3] Email address: ketov@phys.metro-u.ac.jp

1 Introduction

Spontaneously broken global supersymmetry plays important role in brane-world models. D-branes generically break some part of supersymmetry that is, however, still *non-linearly* realised in the effective field theory in the D-brane worldvolume. The effective Lagrangians are highly constrained by the spontaneously broken supersymmetry while in some cases matter couplings can be determined exactly (see e.g., ref. [1] for a recent review).

Spontaneously broken global supersymmetry is always accompanied by a (Nambu-Goldstone-type) fermionic (spin-1/2) particle called Goldstino [2]. The invariant Goldstino action (at least in its minimal form, as an extension of Dirac action) is supposed to be fixed by the spontaneously broken supersymmetry up to field redefinition. Checking the independence of a given Goldstino action upon its origin (i.e. its *universality*) is often a highly non-trivial excercise in practice. In this Letter we give a new relevant example of this universality.

Our paper is organized as follows. In sect. 2 we review the standard *Akulov-Volkov* (AV) action [3] for Goldstino, originating from the universal (coset) formalism of non-linear realizations. In sect. 3 we review the N=1 supersymmetric *Bagger-Galperin* (BG) action [4], originating from the spontaneous partial supersymmetry breaking, N=2 to N=1, in superspace. The new Goldstino action, descending from the BG action, is derived in sect. 4. A connection to the AV action by field redefinition is discussed in sect. 5. Our conclusions are summarized in sect. 6. We use the standard (2-component) notation for spinors in four (Minkowski) spacetime dimensions, as is given e.g., in ref. [5].

2 Akulov-Volkov action

The standard Goldstino action associated with the non-linearly realised supersymmetry is the so-called Akulov-Volkov (AV) action [3]. A supersymmetry transformation is most naturally defined in superspace as a shift of the superspace coordinates,

$$\begin{aligned} x^m \;&\to\; x'^m = x^m + i(\theta\sigma^m\bar{\varepsilon} - \varepsilon\sigma^m\bar{\theta}) \ , \\ \theta \;&\to\; \theta' = \theta + \varepsilon \ , \quad \bar{\theta} \;\to\; \bar{\theta}' = \bar{\theta} + \bar{\varepsilon} \ . \end{aligned} \tag{2.1}$$

The AV Goldstino spinor λ of broken N=1 supersymmetry can be viewed as a superspace hypersurface defined by [6]

$$\theta = -\kappa\lambda(x) \ , \tag{2.2}$$

where the dimensional coupling constant κ of mass dimension -2 has been introduced. The coupling constant κ determines the supersymmetry breaking scale. Requiring the hypersurface (2.2) to be invariant under the transformations (2.1), i.e. $\theta'(x) = \theta(x')$, gives rise to the standard non-linear supersymmetry transformation law [3]

$$\delta_\varepsilon \lambda = \frac{1}{\kappa}\varepsilon + i\kappa(\varepsilon\sigma^m\bar{\lambda} - \lambda\sigma^m\bar{\varepsilon})\partial_m\lambda \ . \tag{2.3}$$

The non-linear transformations (2.3) represent the standard supersymmetry algebra

$$[\delta_{\varepsilon_1}, \delta_{\varepsilon_2}]\lambda = -2i(\varepsilon_1\sigma^m\bar{\varepsilon}_2 - \varepsilon_2\sigma^m\bar{\varepsilon}_1)\partial_m\lambda \ . \tag{2.4}$$

To construct an invariant action one defines

$$\omega_m{}^n = \delta_m{}^n + i\kappa^2(\lambda\sigma^n\partial_m\bar{\lambda} - \partial_m\lambda\sigma^n\bar{\lambda}) \tag{2.5}$$

so that $\det(\omega)$ transforms as a density under the non-linear supersymmetry,

$$\delta_\varepsilon \det(\omega) = -i\kappa\partial_m\left[(\lambda\sigma^m\bar{\varepsilon} - \varepsilon\sigma^m\bar{\lambda})\det(\omega)\right] \ . \tag{2.6}$$

The invariant AV action reads [3]

$$S[\lambda] = -\frac{1}{2\kappa^2}\int d^4x\ \det\omega = -\frac{1}{2\kappa^2}\int d^4x - i\int d^4x\,\lambda\sigma^m\partial_m\bar{\lambda} + \text{interaction terms} \ . \tag{2.7}$$

The AV Lagrangian has, therefore, the non-vanishing vacuum expectation value $-1/2\kappa^2$, while its leading term is given by the Dirac Lagrangian, as it should. The N=1 supersymmetry is entirely realised in terms of a *single* fermionic field $\lambda(x)$. It is possible to construct various supermultiplets of *linear* supersymmetry in terms of the Goldstino field of the *non-linear* supersymmetry, order by order in κ, see e.g., refs. [6, 7]. For a later use, we calculate the leading interaction terms in the AV Lagrangian,

$$\begin{aligned} L_{\rm AV} = &-\frac{1}{2\kappa^2}\det\omega = -\frac{1}{2\kappa^2} - \frac{i}{2}(\lambda\sigma^m\partial_m\bar{\lambda} - \partial_m\lambda\sigma^m\bar{\lambda}) \\ &- \frac{\kappa^2}{4}(\lambda\sigma^n\partial_m\bar{\lambda} - \partial_m\lambda\sigma^n\bar{\lambda})(\lambda\sigma^m\partial_n\bar{\lambda} - \partial_n\lambda\sigma^m\bar{\lambda}) \\ &+ \frac{\kappa^2}{4}(\lambda\sigma^m\partial_m\bar{\lambda} - \partial_m\lambda\sigma^m\bar{\lambda})(\lambda\sigma^n\partial_n\bar{\lambda} - \partial_n\lambda\sigma^n\bar{\lambda}) + \mathcal{O}(\kappa^4) \ . \end{aligned} \tag{2.8}$$

3 Bagger-Galperin action

The D-brane worldvolume effective action necessarily contains the terms describing a selfinteracting abelian vector field A_μ. In the case of a single 'spacetime-filling'

D3-brane, which is relevant here, those (low-energy) terms are given by the (1+3)-dimensional *Born-Infeld* (BI) action [8, 9]

$$S_{\rm BI} = \frac{1}{\kappa^2}\int d^4x\,\left(1-\sqrt{-\det(\eta_{mn}+\kappa F_{mn})}\right)\ , \tag{3.1}$$

where $F_{mn}=\partial_m A_n - \partial_n A_m$ is the abelian field strength. The photon field strength F_{mn} can be extended to the N=1 chiral spinor superfield strength W_α of an N=1 vector (Maxwell) supermultiplet as follows [5]:

$$\begin{aligned} W_\alpha(x,\theta,\bar\theta) \ = \ & \psi_\alpha - i(\sigma^{mn}\theta)_\alpha F_{mn} + \theta_\alpha D - i\theta^2(\sigma^m\partial_m\bar\psi)_\alpha \\ & + i(\theta\sigma^m\bar\theta)\partial_m\psi_\alpha - \tfrac{1}{2}\theta^2(\sigma^m\bar\theta)_\alpha(i\partial_m D - \partial^n F_{mn}) + \tfrac{1}{4}\theta^2\bar\theta^2\Box\psi_\alpha \end{aligned} \tag{3.2}$$

that contains, in addition to $F_{mn}(x)$ its fermionic superpartner (photino) $\psi_\alpha(x)$ and the real auxiliary scalar $D(x)$. We use the notation $\theta^2=\theta^\alpha\theta_\alpha$ for chiral spinors and spinor covariant derivatives (see below), and similarly for the anti-chiral ones. The superfield W_α satisfies the N=1 superspace Bianchi identities [5]

$$\bar D_{\dot\alpha} W_\alpha = 0 \quad \text{and} \quad D^\alpha W_\alpha = \bar D_{\dot\alpha}\bar W^{\dot\alpha}\ , \tag{3.3}$$

where we have introduced the standard N=1 supercovariant derivatives in superspace,

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i(\sigma^m\bar\theta)_\alpha\partial_m\ , \quad \bar D_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - i(\theta\sigma^m)_{\dot\alpha}\partial_m\ , \tag{3.4}$$

obeying the relations

$$\{D_\alpha, D_\beta\} = 0 \quad \text{and} \quad \{D_\alpha, \bar D_{\dot\beta}\} = -2i\sigma^m_{\alpha\dot\beta}\partial_m\ . \tag{3.5}$$

The manifestly N=1 supersymmetric generlization of the BI action (3.1) was found by Bagger and Galperin [4] in the form [4]

$$S_{\rm BG} = \int d^4x\, L_{\rm BG} = \int d^4x\left(\int d^2\theta\, X + \text{h.c.}\right)\ , \tag{3.6}$$

whose N=1 chiral superfield Lagrangian X is determined by the recursive formula [4]

$$X = \frac{\frac{1}{4}W^2}{1+\frac{\kappa^2}{4}\bar D^2\bar X}\ . \tag{3.7}$$

It follows from eqs. (3.6) and (3.7) that

$$L_{\rm BG} = \left[\frac{1}{4}\int d^2\theta\, W^2 + \text{h.c.}\right] + \frac{\kappa^2}{8}\int d^2\theta d^2\bar\theta\, W^2\bar W^2 + \mathcal{O}(\kappa^4)\ , \tag{3.8}$$

[4]See ref. [10] for the manifestly N=2 supersymmetric generalization of the BI action.

whose leading terms describe the N=1 supersymmetric Maxwell Lagrangian, as they should. The exact solution to the non-linear constraint (3.7) is given by [4]

$$X = \frac{1}{4}W^2 - \frac{\kappa^2}{32}\bar{D}^2 \left[\frac{W^2\bar{W}^2}{1 - \frac{1}{2}A + \sqrt{1 + \frac{1}{4}B^2 - A}} \right] , \tag{3.9}$$

where

$$\begin{aligned} A &= -\frac{\kappa^2}{8}(D^2W^2 + \bar{D}^2\bar{W}^2) , \\ B &= -\frac{\kappa^2}{8}(D^2W^2 - \bar{D}^2\bar{W}^2) . \end{aligned} \tag{3.10}$$

As regards natural *non-abelian* N=1 and N=2 supersymmetric extensions of the BI action (3.1), see ref. [11].

By construction the N=1 BI action (3.6) has manifest (linearly realised) N=1 supersymmetry. On the top of it there is another (non-linear) supersymmetry, whose transformation law is given by [4]

$$\delta_\eta W_\alpha = \left(\frac{2}{\kappa} + \frac{\kappa}{2}\bar{D}^2\bar{X} \right) \eta_\alpha + 2i\kappa(\sigma^m\bar{\eta})_\alpha \partial_m X \quad . \tag{3.11}$$

The invariance of the action (3.6) under the transformation (3.11) is highly non-trivial, given the fact that X is a complicated function of W and $\bar{W}$, see eq. (3.9). In addition, eq. (3.11) is fully consistent with the N=1 superfield Bianchi identities (3.3). The invariance of the N=1 superfield BI action (3.6) under the transformation (3.11) is technically a consequence of the fact that [4]

$$\delta_\eta X = \frac{1}{\kappa}W^\alpha\eta_\alpha \quad . \tag{3.12}$$

The bosonic BI action (3.1) is recovered from eq. (3.6) after integrating over θ's and setting $\psi_\alpha = D = 0$. We are going to consider now the purely *fermionic* terms in the N=1 BI action by setting

$$F_{mn} = D = 0 . \tag{3.13}$$

Of course, the truncation (3.13) of the action (3.6) explicitly breaks the linear N=1 supersymmetry. However, it is still consistent with the second *non-linearly* realised supersymmetry (3.11) because that leaves the constraint (3.13) to be invariant, i.e.

$$\delta_\eta \left. (D_\alpha W_\beta)\right|_{F_{mn}=D=0} = 0 \quad , \tag{3.14}$$

where | stands for the first (leading) component of a superfield or an operator. Being subject to extra constraint (3.13), the N=1 supersymmetric BG action (3.6) thus gives rise to a new Goldstino action in terms of the Goldstino spinor field $\psi_\alpha(x)$ alone. To the best of our knowledge, the fermionic terms in the BG action were not investigated in the literature yet.

4 New Goldstino action

The purpose of this section is to derive the new Goldstino action $S[\psi]$ descending from the BG action (3.6) after imposing the constraints (3.13).

The chiral superfield X can be expanded in its field components (ϕ, χ^α, F) as follows:

$$\begin{aligned} X =& \phi + \theta\chi + \theta^2 F + i(\theta\sigma^m\bar{\theta})\partial_m\phi \\ & - \frac{i}{2}\theta^2(\partial_m\chi\sigma^m\bar{\theta}) + \frac{1}{4}\theta^2\bar{\theta}^2\Box\phi \ , \end{aligned} \tag{4.1}$$

so that we have

$$X| = \phi(x), \quad \int d^2\theta\, X = -\frac{1}{4}\, D^2 X\big| = F(x) \ . \tag{4.2}$$

The constraints (3.13) imply

$$\chi_\alpha = D_\alpha X| = 0 \ . \tag{4.3}$$

As is clear from eqs. (3.6) and (4.2), we have

$$S[\psi] = \int d^4x\, L = \int d^4x\, F + \text{h.c.} \tag{4.4}$$

It is not difficult to derive the recursive relation on the field F and its complex conjugate $\bar{F}$ from the recursive relation (3.7) on the superfields X and its complex conjugate $\bar{X}$ by using eqs. (3.13) and (4.1), as well as the identities

$$\begin{aligned} D^2 W_\alpha =& \ 4i\sigma^m_{\alpha\dot{\beta}}\partial_m\bar{W}^{\dot{\beta}} \ , \\ D^2\bar{D}^2\bar{W}^2 =& \ 16\Box\bar{W}^2 \ , \end{aligned} \tag{4.5}$$

and

$$\begin{aligned} D_\alpha X =& -\frac{\frac{1}{2}W^\beta D_\alpha W_\beta}{1+\frac{\kappa^2}{4}\bar{D}^2\bar{X}} + \frac{\frac{i\kappa^2}{4}W^2\sigma^m_{\alpha\dot{\beta}}\bar{D}^{\dot{\beta}}\partial_m\bar{X}}{\left(1+\frac{\kappa^2}{4}\bar{D}^2\bar{X}\right)^2} \ , \\ D^2 X =& \frac{2iW\sigma^n\partial_n\bar{W}}{1+\frac{\kappa^2}{4}\bar{D}^2\bar{X}} - \frac{\kappa^2 W^2\Box\bar{X}}{(1+\frac{\kappa^2}{4}\bar{D}^2\bar{X})^2} \ . \end{aligned} \tag{4.6}$$

As a result, we arrive at a non-linear constraint on $F(\psi,\bar{\psi})$ in the form

$$-4F(1-\kappa^2\bar{F})^2 = 2i(\psi\sigma^n\partial_n\bar{\psi})(1-\kappa^2\bar{F}) - \frac{1}{4}\kappa^2\psi^2\Box\left[\frac{\bar{\psi}^2}{1-\kappa^2 F}\right] \tag{4.7}$$

and its complex conjugate. As will be shown in Sect. 5, despite of the apparent presence of higher derivatives in eq. (4.7), this equation does not imply the equations of motion. Instead, it should be considered as the off-shell recursive relation on the

Lagrangian $L(\psi,\bar{\psi}) = 2\mathrm{Re}(F)$. Equation (4.7) fully determines F and, hence, the action (4.4) in terms of ψ, $\bar{\psi}$ and their spacetime derivatives.

The leading term in L is given by the Dirac Lagrangian, as it should,

$$F_0 \equiv F|_{\kappa^2=0} = -\frac{i}{2}\psi\sigma^m\partial_m\bar{\psi} \equiv \frac{1}{2}L_0 \quad . \tag{4.8}$$

The exact (i.e. to all orders in κ^2) solution to the non-linear constraint (4.7) descends from the superfield solution (3.9). A straightforward (albeit tedious) calculation yields

$$\begin{aligned}
F = \frac{1}{2}L_0 &+ \frac{\frac{\kappa^2}{8}\left(4\left|L_0\right|^2 + \psi^2\Box\bar{\psi}^2\right)}{1-\frac{1}{2}A_0+\sqrt{1+\frac{1}{4}B_0^2-A_0}} \\
&+ \frac{\frac{\kappa^4}{16}\bar{L}_0\psi^2\Box\bar{\psi}^2}{\left(1-\frac{1}{2}A_0+\sqrt{1+\frac{1}{4}B_0^2-A_0}\right)^2}\left[1+\frac{1+\frac{1}{2}B_0}{\sqrt{1+\frac{1}{4}B_0^2-A_0}}\right] \\
&+ \frac{\frac{\kappa^4}{16}\left\{\psi^2\bar{\psi}^2\Box L_0 + 2\psi^2\partial^m\bar{\psi}^2\partial_m L_0 + L_0\bar{\psi}^2\Box\psi^2\right\}}{\left(1-\frac{1}{2}A_0+\sqrt{1+\frac{1}{4}B_0^2-A_0}\right)^2}\left[1+\frac{1-\frac{1}{2}B_0}{\sqrt{1+\frac{1}{4}B_0^2-A_0}}\right] \\
&+ \frac{\frac{\kappa^6}{64}\psi^2\bar{\psi}^2\Box\psi^2\Box\bar{\psi}^2}{\left(1-\frac{1}{2}A_0+\sqrt{1+\frac{1}{4}B_0^2-A_0}\right)^3}\left[1+\frac{1+\frac{1}{2}B_0}{\sqrt{1+\frac{1}{4}B_0^2-A_0}}\right]\left[1+\frac{1-\frac{1}{2}B_0}{\sqrt{1+\frac{1}{4}B_0^2-A_0}}\right] \\
&+ \frac{\frac{\kappa^6}{128}\psi^2\bar{\psi}^2\Box\psi^2\Box\bar{\psi}^2}{\left(1-\frac{1}{2}A_0+\sqrt{1+\frac{1}{4}B_0^2-A_0}\right)^2}\frac{1}{\sqrt{1+\frac{1}{4}B_0^2-A_0}}\left[1+\frac{1-\frac{1}{4}B_0^2}{1+\frac{1}{4}B_0^2-A_0}\right] \\
&- \frac{\frac{\kappa^6}{32}\psi^2\bar{\psi}^2\partial^m L_0\partial_m L_0}{\left(1-\frac{1}{2}A_0+\sqrt{1+\frac{1}{4}B_0^2-A_0}\right)^2}\frac{1}{\sqrt{1+\frac{1}{4}B_0^2-A_0}}\left[1-\left(\frac{1-\frac{1}{2}B_0}{\sqrt{1+\frac{1}{4}B_0^2-A_0}}\right)^2\right] \\
&+ \frac{\frac{\kappa^6}{16}\psi^2\bar{\psi}^2\partial^m L_0\partial_m L_0}{\left(1-\frac{1}{2}A_0+\sqrt{1+\frac{1}{4}B_0^2-A_0}\right)^3}\left[1+\frac{1-\frac{1}{2}B_0}{\sqrt{1+\frac{1}{4}B_0^2-A_0}}\right]^2 \quad ,
\end{aligned} \tag{4.9}$$

where we have used eq. (3.10) and the notation

$$A_0 = A| = 2\kappa^2\mathrm{Re}(L_0) \ , \quad B_0 = B| = 2\kappa^2 i\mathrm{Im}(L_0) \ . \tag{4.10}$$

The most important property of the action (4.4) is its invariance under the following (rigid) non-linear supersymmetry transformations,

$$\delta_\eta\psi_\alpha = \left(\frac{2}{\kappa} - 2\kappa\bar{F}\right)\eta_\alpha + \frac{i}{2}\kappa(\sigma^m\bar{\eta})_\alpha\partial_m\left(\frac{\psi^2}{1-\kappa^2\bar{F}}\right) \ , \tag{4.11}$$

descending from eq. (3.11). In particular, we have $\langle 0|\delta_\eta \psi_\alpha|0\rangle = \frac{2}{\kappa}\eta^\alpha \neq 0$, i.e. the vacuum is not supersymmertric. This fact allows us to call $\psi_\alpha(x)$ the Goldstino field, so that we can identify the action (4.4) as the Goldstino action associated with the spontaneous N=1 supersymmetry breaking.

Despite of the apparent presence of many square roots in the Lagrangian (4.9), it is, in fact, *polynomial* in ψ and $\bar{\psi}$ due to their anticommutativity that implies the nilponency conditions

$$\psi^3 = \bar{\psi}^3 = L_0^3 = \bar{L}_0^3 = A_0^5 = B_0^5 = 0 \quad . \tag{4.12}$$

5 Relation between the AV and BG actions

The new Goldstino action $S[\psi]$ descending from the BG action (see sect. 4) seems to be very different from the standard AV action $S[\lambda]$ (see sect. 2). For example, the action $S[\lambda]$ has only the first-order spacetime derivatives of λ, whereas the action $S[\psi]$ has the second order derivatives of ψ also.

Both actions can be used as the Goldstino action because they are invariant under the corresponding non-linear supersymmetry transformations having inhomogeneous shifts proportional to the anticommuting spinor parameter η.

Whatever the reason for a spontaneous global symmetry breakdown, the broken symmetry is supposed to fix the invariant (minimal) Goldstone action up to field redefinition. The anticipated universality of the Goldstino action gives us a good reason to suspect that the actions $S[\lambda]$ and $S[\psi]$ may be equivalent up to a field redefinition. We are now going to check this conjecture up to the order κ^2.

The difference between the actions $S[\lambda]$ and $S[\psi]$ can be thought of as the direct consequence of the difference between the corresponding supersymmetry transformation laws in eqs. (2.3) and (4.11), respectively. The transformation law (2.3) originated from the general (coset) approach to non-linear realizations of N=1 supersymmetry, whereas the BG-type transformation law (4.11) appeared out of the context of partial spontaneous supersymmetry breaking, N=2 to N=1, when first making manifest the unbroken N=1 supersymmetry of the corresponding Goldstone-Maxwell action. Hence, the field redefinition in question can be found by applying the known relation between linear and non-linear realizations of supersymmetry [6, 12].

The structure of the supersymmetry transformations (3.11) and (3.12) implies that the fields $\phi(x) = X|$ and $\psi_\alpha(x) = W_\alpha|$ are, in fact, the components of an N=1

chiral superfield with respect to the second N=1 supersymmetry,

$$X(x,\zeta,\bar{\zeta}) = \phi + \frac{1}{\kappa}\psi\zeta + \left(\frac{1}{\kappa^2} - \bar{F}\right)\zeta^2 + i(\zeta\sigma^m\bar{\zeta})\partial_m\phi \\ - \frac{i}{2\kappa}\zeta^2(\partial_m\psi\sigma^m\bar{\zeta}) + \frac{1}{4}\zeta^2\bar{\zeta}^2\Box\phi \ , \tag{5.1}$$

since eqs. (3.11) and (3.12) then follow from eq. (2.1) in N=1 superspace $(x,\zeta,\bar{\zeta})$,

$$\delta x^m = i(\zeta\sigma^m\bar{\eta} - \eta\sigma^m\bar{\zeta}) \ , \quad \delta\zeta = \eta \ , \quad \delta\bar{\zeta} = \bar{\eta} \ . \tag{5.2}$$

The procedure of passing to the standard non-linear realization of supersymmetry, in the case of partial spontaneous supersymmetry breaking N=2 to N=1, was formulated in ref. [12]. It equally applies to our case of spontaneous supersymmetry breaking N=1 to N=0. The idea is to consider the *finite* η-transformations of the fields ϕ and ψ_α , which are generated by the infinitesimal supersymmetry transformations (3.11) and (3.12),

$$\begin{pmatrix} \tilde{\phi}(\eta) \\ \tilde{\psi}_\alpha(\eta) \end{pmatrix} = \left(1 + \delta_\eta + \frac{1}{2!}\delta_\eta^2 + \frac{1}{3!}\delta_\eta^3 + \frac{1}{4!}\delta_\eta^4\right)\begin{pmatrix} \phi \\ \psi_\alpha \end{pmatrix} \ , \tag{5.3}$$

and then replace the anticommuting parameters $(\eta,\bar{\eta})$ by the AV fermions, by using the standard rule (2.2), [5]

$$\eta_\alpha \to -\frac{\kappa}{2}\lambda_\alpha \ , \quad \bar{\eta}_{\dot{\alpha}} \to -\frac{\kappa}{2}\bar{\lambda}_{\dot{\alpha}} \ . \tag{5.4}$$

The composite fields $\tilde{\phi}(\lambda)$ and $\tilde{\psi}_\alpha(\lambda)$ transform (non-linearly) homogeneously under the η-transformations [12],

$$\delta_\eta\begin{pmatrix} \tilde{\phi}(\lambda) \\ \tilde{\psi}_\alpha(\lambda) \end{pmatrix} = \frac{i}{2}\left(\lambda^\alpha\bar{\eta}^{\dot{\alpha}} - \eta^\alpha\bar{\lambda}^{\dot{\alpha}}\right)\partial_{\alpha\dot{\alpha}}\begin{pmatrix} \tilde{\phi}(\lambda) \\ \tilde{\psi}_\alpha(\lambda) \end{pmatrix} \ , \tag{5.5}$$

so that the constraints

$$\tilde{\phi}(\lambda) = \tilde{\psi}_\alpha(\lambda) = 0 \tag{5.6}$$

are invariant under the non-linear supersymmetry. Equations (5.6) give us the desired relation between the spinor fields $\psi(x)$ and $\lambda(x)$ in the closed (though rather implicit) form (*cf.* refs. [7, 12]):

$$0 = \phi - \frac{\lambda^2}{4}\left(1 - \kappa^2\bar{F}\right) - \frac{i\kappa^2}{4}\left(\lambda\sigma^m\bar{\lambda}\right)\partial_m\phi \\ - \frac{i\kappa^2}{8}\lambda^2\left(\partial_m\psi\sigma^m\bar{\lambda}\right) + \frac{3\kappa^2}{64}\lambda^2\bar{\lambda}^2\Box\phi \tag{5.7}$$

[5] We rescaled λ by the factor of 1/2 for a later convenience.

and

$$0 = \psi_\alpha - \lambda_\alpha(1-\kappa^2\bar{F}) - i\kappa^2(\sigma^m\bar{\lambda})_\alpha\partial_m\phi - \frac{i\kappa^2}{4}\left(\partial_m\psi\sigma^m\bar{\lambda}\right)\lambda_\alpha + \frac{i\kappa^2}{4}(\sigma^m\bar{\lambda})_\alpha(\lambda\partial_m\psi)$$
$$+ \frac{i\kappa^4}{8}\lambda^2(\sigma^m\bar{\lambda})_\alpha\partial_m\bar{F} - \frac{\kappa^4}{8}\lambda_\alpha\bar{\lambda}^2\Box\phi - \frac{\kappa^4}{64}\lambda^2\bar{\lambda}^2\Box\psi_\alpha \ , \tag{5.8}$$

where the field F is given by eq. (4.7) or (4.9). [6] Equations (5.7) and (5.8) can be used to unambiguously calculate ψ as a function of λ order-by-order in κ^2, after eliminating the auxiliary fields ϕ and F. The invariant (under the non-linear supersymmetry) relation between ψ and λ in eq. (5.8) is not universal because the Goldstino Lagrangian $L = 2\mathrm{Re}(F)$ enters eq. (5.8) as the auxiliary field, see eq. (4.4).

When keeping only the terms of order κ^0 and κ^2, we find the leading and subleading terms in the action (4.4) as follows:

$$\begin{aligned} L = F + \bar{F} = &-\frac{i}{2}\psi\sigma^n\partial_n\bar{\psi} + \frac{i}{2}\partial_n\psi\sigma^n\bar{\psi} + \frac{\kappa^2}{2}(\psi\sigma^n\partial_n\bar{\psi})(\partial_m\psi\sigma^m\bar{\psi}) \\ &\frac{\kappa^2}{16}\psi^2\Box\bar{\psi}^2 + \frac{\kappa^2}{16}\bar{\psi}^2\Box\psi^2 + \mathcal{O}(\kappa^4) \ . \end{aligned} \tag{5.9}$$

Having substituted eqs. (5.7) and (5.8) into eq. (5.9) up to the given order in κ^2, we recovered eq. (2.8) by using Fierzing identities and integration by parts. This pattern may persist to all orders in κ^2, though we still cannot exclude a possible higher-order superinvariant, with at least four spacetime derivatives, as a non-trivial difference between $S[\psi(\lambda)]$ and $S[\lambda]$.

6 Conclusion

The non-linear realizations formalism gives rise to the AV action $S[\lambda]$ for Goldstino without higher derivatives. When one wants to assign Goldstino to a vector (Maxwell) supermultiplet (thus identifying Goldstino and photino), in the context of partial spontaneous supersymmetry breaking, one gets another Goldstino action $S[\psi]$ that has higher (second-order) derivatives. The new action $S[\psi]$ appears to be equivalent to the AV action $S[\lambda]$ up to a field redefinition and integration by parts. By providing the field redefinition in question we verified the universality of Goldstino action up to the terms of order κ^2. The proposed on-shell equivalence of the actions $S[\lambda]$ and $S[\psi]$ is highly non-trivial because of the complicated relation between the Goldstino fields λ and ψ, described by eqs. (4.9), (5.7) and (5.8).

[6] Equation (5.8) was used in deriving eq. (5.7) from eq. (5.3).

Acknowledgements

The authors are grateful to Augusto Sagnotti, whose question led to this investigation, Jim Gates Jr. and Satoru Saito for useful discussions, and the referee for his careful reading of our manuscript. One of the authors (S.V.K.) would like to thank the Center for String and Particle Theory (CSPT) at the University of Maryland in College Park (USA) for kind hospitality during some part of this work.

References

[1] M. Klein, Phys. Rev. **D66** (2002) 055009 [hep-th/0205300]

[2] S. Weinberg, *The Quantum Theory of Fields*, Vol. III, Cambridge University Press, 2000

[3] D. V. Volkov and V. P. Akulov, JETP Lett. **16** (1972) 438; Phys. Lett. **46B** (1973) 109

[4] J. Bagger and A. Galperin, Phys. Rev. **D 55** (1997) 1091 [hep-th/9608177]

[5] J. Wess and J. Bagger, *Supersymmetry and Supergravity*, 2nd edition, Princeton University Press, 1992

[6] E. A. Ivanov and A. A. Kapustnikov, J. Phys. **A11** (1978) 2375; Nucl. Phys. **B333** (1990) 439

[7] M. Roček, Phys. Rev. Lett. **41** (1978) 451

[8] M. Born and L. Infeld, Proc. Roy. Soc. Lond. **A144** (1934) 425

[9] R. G. Leigh, Mod. Phys. Lett. **A4** (1989) 2767

[10] S. V. Ketov, Mod. Phys. Lett. **A14** (1999) 501 [hep-th/9809121]; Nucl. Phys. **B553** (1999) 250 [hep-th/9812051]; Class. and Quantum Grav. **17** (2000) L91 [hep-th/0005126]

[11] S. V. Ketov, Phys. Lett. **491B** (2000) 207 [hep-th/0005265]

[12] E. A. Ivanov, Theor. Math. Phys. **129** (2001) 1543 [hep-th/0105210].